Multilateral market power in input-output networks

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Workshop on Market Power in Supply Chains

Question

Two features of today's economies:

- long, interconnected **supply chains**;
- market power is important both on **output** and **input markets**

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- long, interconnected **supply chains**;
  (Berlingieri (2013), Alfaro et al. (2019), ...)

- market power is important both on **output** and **input markets**
(De Loecker and Eeckhout (2020), Morlacco (2020), Berger et al. (2022) ...)

- What is the effect of market power both on inputs and outputs?
- How does the network affect market power?

This paper:
Oligopoly with firm-to-firm trade and **endogenous** market power:

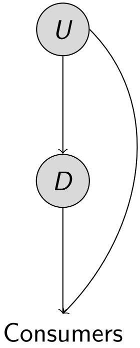
- all firms have market power on **both** input and output markets;
- both **size** and **division** of surplus are endogenous.

Main results

1. **Effect of multi-lateral market power** $\rightarrow$ focus of today
   - we recover standard models as special cases:
     e.g. unilateral market power;
   - multilateral market power increases inefficiencies.

2. **Relation between network and market power**
   Markups/markdowns are related to Bonacich centrality in the **goods network**;

The model through a simple example



- an **upstream** U and a **downstream** layer D;
- network is **given**;

Firms play a **simultaneous game** in which:

- firm U commits to a **supply function** S_U;
- firm D commits to a **supply function** S_D and a **demand function** D_D;
  - s.t. **technology constraint**

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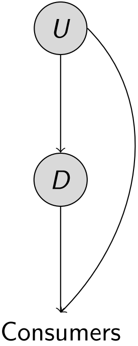

Parametric assumptions

- the **technology** is linear: q_i produced from q_i of input;

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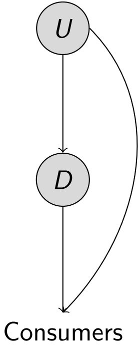

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- the **technology** is linear: q_i produced from q_i of input;
- **consumers**:
  - price takers;
  - Consume both goods:

$$c_D = A_D - p_D$$
$$c_U = A_U - p_U$$

- labor market is competitive (w taken as given);
  this presentation: set $w = 0$ for simplicity.

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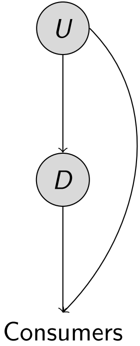

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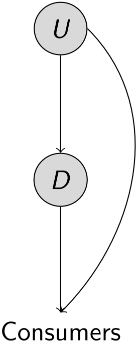
- labor market is competitive (w taken as given);
  this presentation: set $w = 0$ for simplicity.
- Firms restricted to **linear schedules**:
  - $S_U = B_U p_U$;
  - $D_D = B_D(p_D - p_U) = S_D$;

The model through a simple example



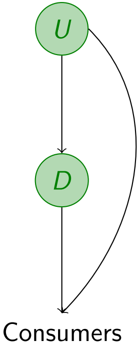
Payoffs:

- $\pi_D = \underbrace{p_D S_D}_{\text{Revenues}} - \underbrace{p_U D_D}_{\text{Intermediate input costs}}$
- $\pi_U = p_U S_U$

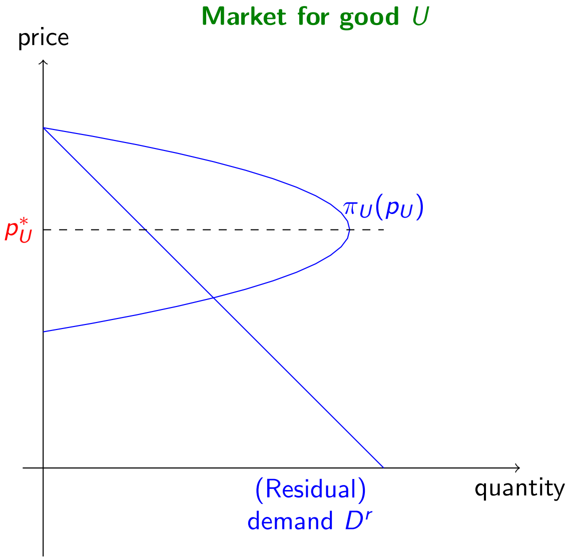
where p_D, p_U solve the **market clearing conditions**:

$$A_D - p_D = S_D$$
$$D_D + A_U - p_U = S_U$$

Intuition



U
D
Consumers



Market for good U
price
p_U^*
$\pi_U(p_U)$
quantity
(Residual)
demand D^r

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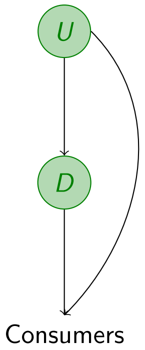
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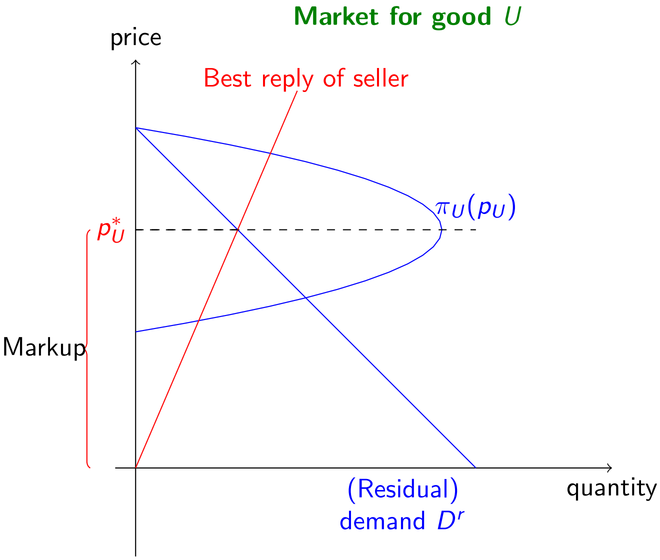
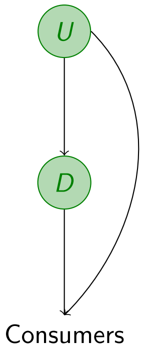

Market for good U
price
Best reply of seller
$\pi_U(p_U)$
p_U^*
Markup
(Residual)
demand D^r
quantity

Intuition



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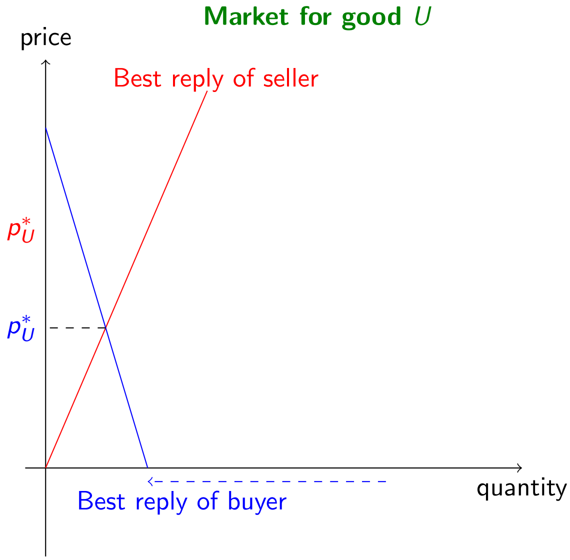

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Best reply of seller
p_U^*
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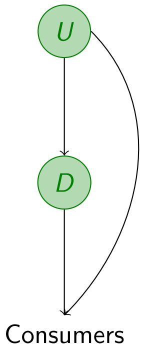
Best reply of buyer

Intuition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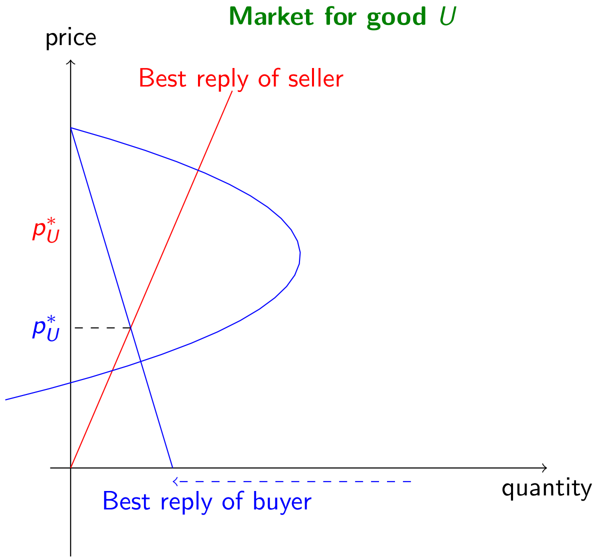
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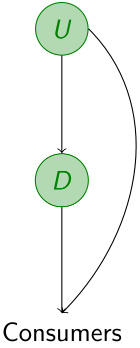

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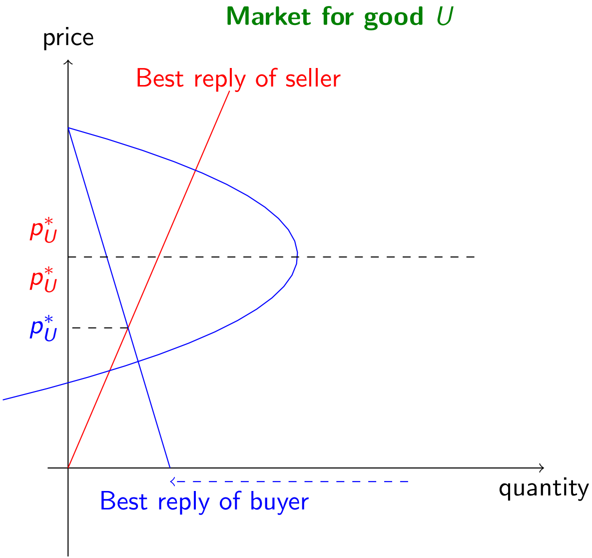

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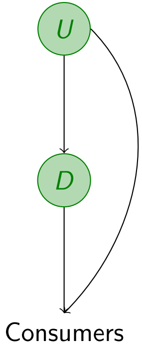

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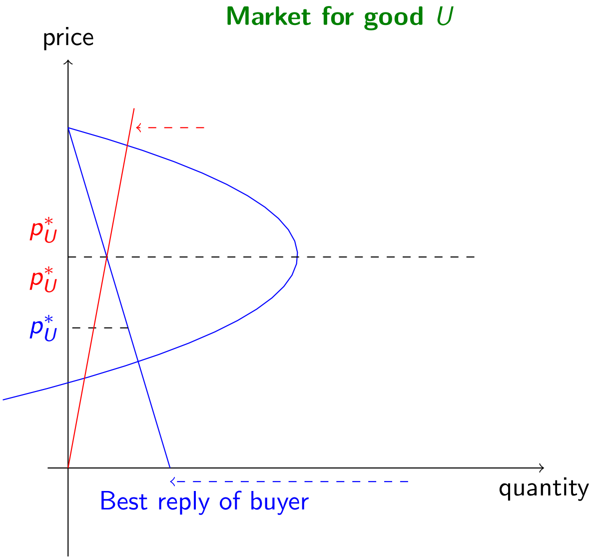

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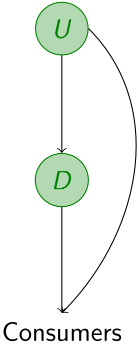

In equilibrium **both** markups and markdown in every intermediate market.

Key mechanism: **strategic complementarity**

- to raise markup, set a **smaller** supply slope;
- smaller slope $\implies$ smaller elasticity of supply;
- $\implies$ customer increases markdown, lowering demand slope;
- $\implies$ smaller elasticity of demand $\implies$ higher markup.

Solution

Solve market clearing eqs. for (inverse) **residual demand and supply**: $p_{D,D}(q)$, $p_{D,U}(q)$.

Just take the FOC:

$$S_U(p_U) = \left(-\frac{\partial p_{U,U}}{\partial q_U}\right)^{-1} p_U$$

$$S_D(p_D, p_U) = \left(\frac{\partial p_{D,D}}{\partial q_D} - \frac{\partial p_{D,U}}{\partial q_D}\right)^{-1} (p_D - p_U)$$

Solution

Solve market clearing eqs. for (inverse) **residual demand and supply**: $p_{D,D}(q)$, $p_{D,U}(q)$.

Just take the FOC:

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This yields equilibrium equations in B_U, B_D. Solution

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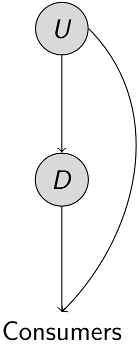
Linear equilibrium survives uncertainty ("productivity shocks") as in Klemperer and Meyer ('89)

Literature



Common approaches:

- market power on one side: **inputs** or **output**;
  Carvalho et al. (WP), Grassi (WP), Baqaee and Farhi (2019)
  Salinger (1990), Ordover et al. (1990)
- exogenous barganing weights:
  Collard-Wexler et al. (2019), **Acemoglu and Tahbaz-Salehi (2022)**
  Alviarez et al. (2023)

Competition in **supply and demand functions**:

Klemperer and Meyer (1989), Vives (2011)
Malamud and Rostek (2017)

more

Multilateral market power

What if instead firms are **price-takers** on input markets?
(="unilateral market power")

Assume $\frac{\partial p_{U,D}}{\partial q_D} = 0$.

- a standard sequential monopoly a' la Spengler (1950).

Multilateral market power

What if instead firms are **price-takers** on input markets?
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Assume $\frac{\partial p_{U,D}}{\partial q_D} = 0$.

- a standard sequential monopoly a' la Spengler (1950).

The fixed point equations become:

$$B_U = \left(-\frac{\partial p_{U,U}}{\partial q_U}\right)^{-1} \quad B_D = \left(\frac{\partial p_{D,D}}{\partial q_D} - \cancel{\frac{\partial p_{D,U}}{\partial q_D}}\right)^{-1}$$

$\implies$ B_D is **larger**: by strategic complementarity, in equilibrium, also B_U.

$\implies$ prices are **lower**.

General networks

In a general network:

- schedule have coefficient **matrix** B_i;
- price impact is a matrix $\Lambda_i = \partial \boldsymbol{p}_i / \partial \boldsymbol{q}_i$;
- Input substitutes/complements: quadratic “handling costs” for labor;
- a vector of markups-markdowns: $\boldsymbol{\mu}_i$.
- Caveat: a non-trivial linear equilibrium exists if at least 3 “agents” trade each good.

The effect of multilateral market power

Theorem
Consider an input-output network in which there is a unique final good:

The effect of multilateral market power

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Consider an input-output network in which there is a unique final good:

1. *If firms are price-takers on input markets, the final price goes down;*

The effect of multilateral market power

Theorem

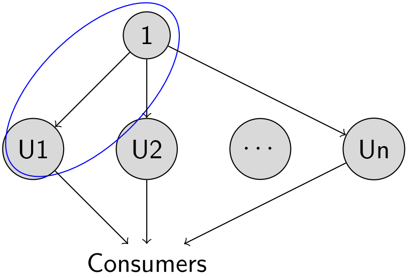
Consider an input-output network in which there is a unique final good:

1. *If firms are price-takers on input markets, the final price goes down;*
2. *If firms neglect their price impact on markets not directly connected, the final price goes down.*

Application: evaluation of a vertical merger

Consider a merger between firm 1 and firm $U1$.

Assume that if merged firm does not sell to others, they all close.



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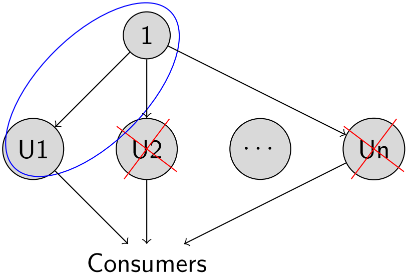

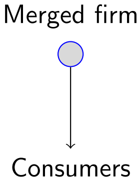

Standard trade-off **foreclosure** vs **less double marginalization**.

There is a range of n such that merger is:

- **welfare improving** if multilateral market power;
- **welfare decreasing** if unilateral.

Discussion

In general production networks:

- who sets/affects which price?
- a modeling assumption that affects the results!

With S&D equilibrium:

- firms affect prices in all markets, in an endogenous way;
- firms are symmetric, but for **network position** and **technology**.

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- firms are symmetric, but for **network position** and **technology**.

Supply and demand functions:

- physically used in finance, electricity auctions;
- here: any arrangement (contractual, managerial) that specify how firm reacts to different conditions in the market.

General mechanism

The price impact matrix Λ_i is a map $B_1, \ldots, B_N \to \Lambda_i(B)$, such that:

1. Λ_i is positive definite;
2. Λ_i is increasing in the psd ordering in each B_j for $j \neq i$.

Perfect competition, Cournot, Bertrand can all be embedded as assumptions on the price impact.

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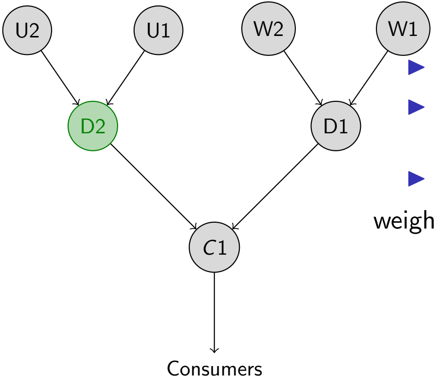
Assuming "No price impact on inputs" means that in the psd order:

$$\Lambda^{\text{multilateral}}(B) > \Lambda^{\text{unilateral}}(B)$$

The goods network

The price impact Λ depends on the **goods network** relative to a firm.
Here: a tree with 4 goods: U, D, W and C. Focus on firm $D2$:



Adjacency matrix

- the nodes are the goods;
- two goods linked if at least a firm trades both, **except** $D2$;
- suppose all coefficients $B = 1$. Then:

weight on (i, j) link$=$

$$\frac{\#\text{agents trading both}}{\sqrt{\#\text{agents trading } i \#\text{agents trading } j}}$$

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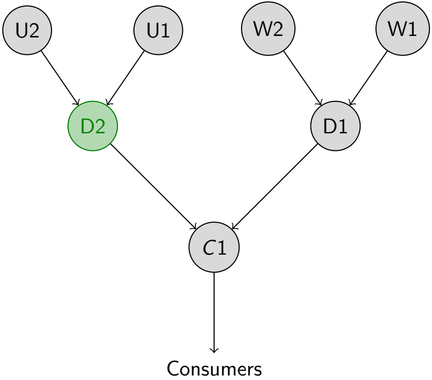

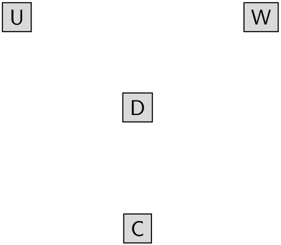

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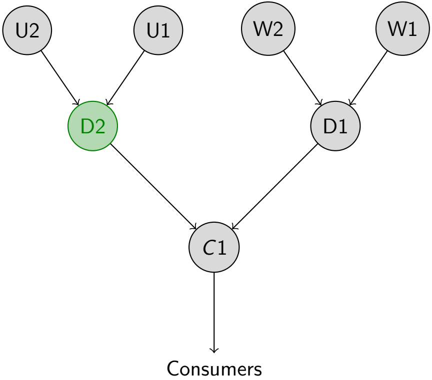

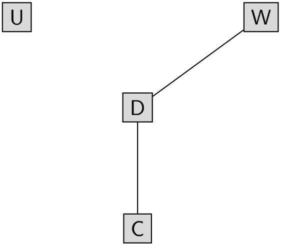

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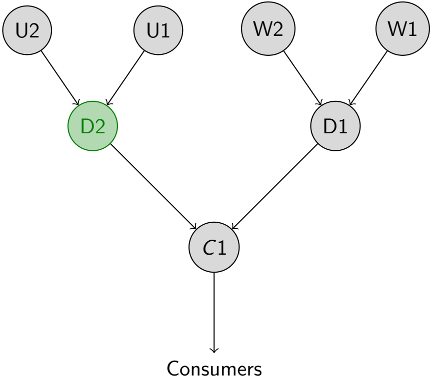

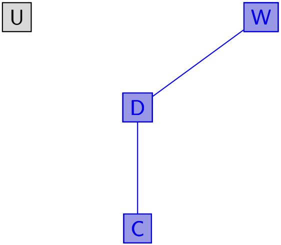

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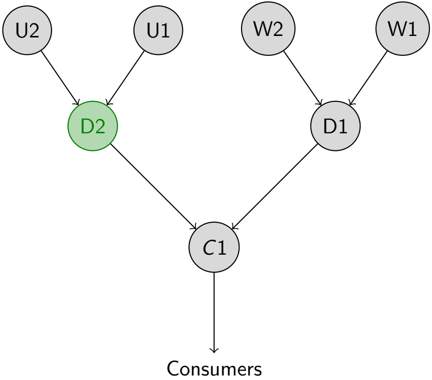

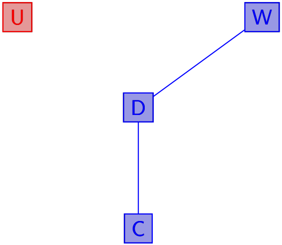

Adjacency matrix

How the network affects market power

Theorem

In equilibrium:

1.

$$\Lambda_{i,gg} = \Lambda_{i,gg}^{no\ network} L_{i,g}$$

- $\Lambda_{i,gg}^{no\ network}$ *is the price impact on good g due only to direct connections;*
- $L_{i,g} =$ *the number of* **cycles** *out of good g in the goods network relative to i.*

How the network affects market power

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    - $L_{i,g}$ = *the number of* **cycles** *out of good* g *in the goods network relative to* i.

2. $\boldsymbol{\mu}_{i,g}$ *counts the number of* **direct and indirect paths** *in the goods network relative to firm* i, *from* g *to each other good traded by* i *(properly weighted)*

    - *an analog of* **Bonacich centrality** *of good* g, *restricted to the neighborhood of* i.

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   - *an analog of* **Bonacich centrality** *of good g, restricted to the neighborhood of i.*

Conclusion

Key messages:

- competition in S&D schedules useful to model multilateral market power:
  - can deal with general firm to firm networks;
  - "easy" to embed some standard assumptions, for comparison.

- allowing for multi-sided market power can change implications for:
  - quantification of distortions;
  - welfare impact of horizontal and vertical mergers;
  - diffusion of productivity changes (in the paper)

Equilibrium in the example

$$B_U^* = 1 + \frac{B_D^*}{B_D^* + 1}$$
$$B_D^* = \left(1 + \frac{1}{B_U^*}\right)^{-1}.$$

In this case, can be solved analytically: $B_D^* = 1/\sqrt{2}$, $B_U^* = \sqrt{2}$. Back

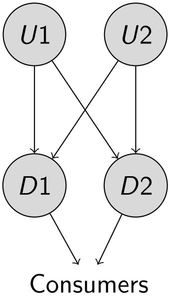
Adjacency matrix for the supply chain

$$M = \begin{pmatrix} B_c + n_1 B_1 & -n_1 B_1 & 0 \\ -n_1 B_1 & n_2 B_2 + n_1 B_1 & -n_2 B_2 \\ 0 & -n_2 B_2 & n_2 B_2 + n_3 B_3 \end{pmatrix}$$

$$G_2 = \begin{pmatrix} 0 & \frac{n_1 B_1}{\sqrt{B_c + n_1 B_1}\sqrt{(n_2-1)B_2 + n_1 \ldots}} \\ \frac{n_1 B_1}{\sqrt{B_c + n_1 B_1}\sqrt{(n_2-1)B_2 + n_1 B_1}} & 0 \\ 0 & \frac{(n_2-1)B_2}{\sqrt{(n_2-1)B_2 + n_1 B_1}\sqrt{(n_2-1)B_2 \ldots}} \end{pmatrix}$$

back

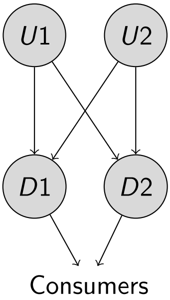
The goods network

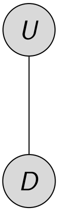


The price impact Λ_i(=inverse slope of the residual demand) depends on the **goods network** relative to firm i:

- the nodes are the goods;
- two goods are linked if there is at least a firm trading both, **apart from** i;
- the links weights depend on the coefficient matrices, **excluding** firm i.

The goods network





The good network here is connected:

- firm $D2$ still connects goods U and D.
- but the **weights** are affected by which firm is considered.

tree network

Size and split of the surplus for the line

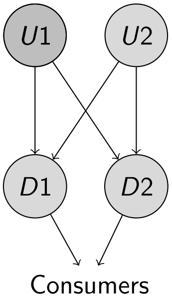
We can express the total profit and the shares as functions of slopes:

$$n_U\pi_U + n_D\pi_D = \frac{A_c\left(\frac{1}{B_U} - \frac{1}{2n_U} + \frac{1}{B_D} - \frac{1}{2n_D}\right)}{B_c\left(\frac{1}{n_U B_U} + \frac{1}{n_D B_D}\right) + 1}$$

$$\frac{n_D\pi_D}{n_D\pi_D + n_U\pi_U} = \frac{\frac{1}{B_D} - \frac{1}{2n_D}}{\frac{1}{B_U} - \frac{1}{2n_U} + \frac{1}{B_D} - \frac{1}{2n_D}}$$

Wrt Nash bargaining both the **size** and **split** of surplus are endogenized.

Neglect higher order network effects



So far: firms (say $U1$) internalize the pass-through of price changes through **all** the network.

Polar case with respect to many macro models: (Grassi (2019), Baqaee (2018), ...)

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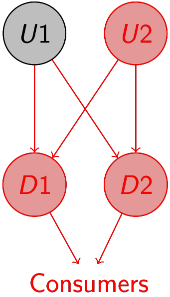

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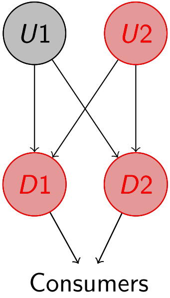

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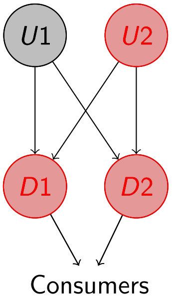

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Compare with firms that internalize only immediate neighbors, that is have **no price impact on other markets**

- **residual demand** (and supply) **steeper** $\Longrightarrow$ lower final price. In general

A literature tries to quantify the distortions due to market power: (Ederer and Pellegrino (WP), Baqaee and Farhi (2020),...)

The impact of rigidities can be **arbitrarily large**:

- In a line network of length N, with 2 firms per layer, we can prove that:

$$\lim_{N \to \infty} \frac{\text{Welfare}^{global}}{\text{Welfare}^{local}} = 0$$

Formal construction of the adjacency matrix

Example: a network of 3 goods: 1, 2 and 3.

The market clearing conditions are a linear system:

$$\begin{pmatrix} M_{11} & M_{12} & 0 \\ M_{12} & M_{22} & M_{23} \\ 0 & M_{23} & M_{33} \end{pmatrix} \begin{pmatrix} p_1 \\ p_2 \\ p_3 \end{pmatrix} = \begin{pmatrix} A_1 \\ 0 \\ 0 \end{pmatrix}$$

Suppose all coefficients B_i are equal to 1.

- M_{gg} counts number of firms **buying or selling** g;
- $-M_{gh}$ counts the number of firms buying or selling **both** g **and** h.

Back Details

Formal construction of the adjacency matrix

We want the good network relative to i. Say firm i buys 3 and outputs 2.

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Normalize by the diagonal D_i:

$$\begin{pmatrix} 1 & \frac{M_{12}}{\sqrt{M_{11}(M_{22}-1)}} & 0 \\ \frac{M_{12}}{\sqrt{M_{11}(M_{22}-1)}} & 1 & \frac{M_{23}-1}{\sqrt{M_{33}(M_{22}-1)}} \\ 0 & \frac{M_{23}-1}{\sqrt{M_{33}(M_{22}-1)}} & 1 \end{pmatrix}$$

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$$D_i' M D_i = Id - \begin{pmatrix} 0 & -\frac{M_{12}}{\sqrt{M_{11}(M_{22}-1)}} & 0 \\ -\frac{M_{12}}{\sqrt{M_{11}(M_{22}-1)}} & 0 & -\frac{M_{23}-1}{\sqrt{M_{33}(M_{22}-1)}} \\ 0 & -\frac{M_{23}-1}{\sqrt{M_{33}(M_{22}-1)}} & 0 \end{pmatrix}$$

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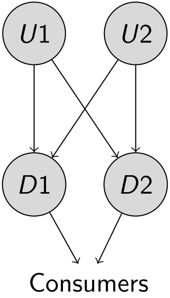
$$G_i = \begin{pmatrix} 0 & -\frac{M_{12}}{\sqrt{M_{11}(M_{22}-1)}} & 0 \\ \frac{M_{12}}{\sqrt{M_{11}(M_{22}-1)}} & 0 & \frac{M_{23}-1}{\sqrt{M_{33}(M_{22}-1)}} \\ 0 & \frac{M_{23}-1}{\sqrt{M_{33}(M_{22}-1)}} & 0 \end{pmatrix}$$

- M^{-1} has the form of a "Leontief inverse";
- The weight $-\sqrt{\frac{M_{12}}{M_{11}}}\sqrt{\frac{M_{12}}{M_{11}M_{33}}}$ represents the geometric average of the fraction of firms trading **both** goods over the firms trading **each**.

Back Details

In which layer market power is stronger?



- markups increasing **upstream**, markdowns **downstream**.

What is the balance?

e.g. think about a competition authority that wants to evaluate interventions.

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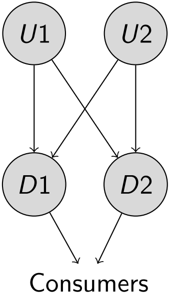


- markups increasing **upstream**, markdowns **downstream**.

What is the balance?

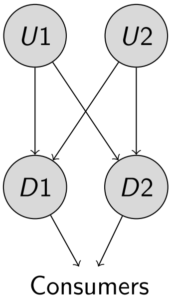
e.g. think about a competition authority that wants to evaluate interventions.

For **this network** the two forces exactly counterbalance each other:

- profits are the same in each layer;
- if we compute the welfare loss from an horizontal merger, they are also **the same**.

Mergers-general Trees

The effect of multilateral market power in a symmetric supply chain



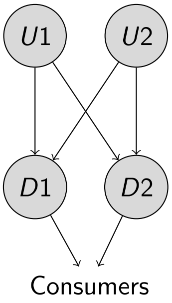
If shut down market power on inputs :

- only markups remain, increasing **upstream**;

If shut down market power on outputs:

- only mark**down**s remain, increasing **downstream**;

The effect of multilateral market power in a symmetric supply chain



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Sequential monopoly model gives analogous results.

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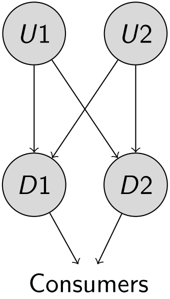

In general production networks:

- who sets/affects which price?
- a modeling assumption that affects the results!

With S&D equilibrium:

- firms take **simultaneously** into account upstream and downstream pass-through
- firms are symmetric, but for **network position** and **technology**.

Technology

To analyze general networks, we need to generalize the technology.

From $q_{i1}, \ldots, q_{in}$ inputs, firm i:

- produce $q_i = \sum_g \omega_{ig} q_{ig}$;
- using labor: $\ell^H(q_{i1}, \ldots, q_{in}) = \frac{1}{2k_i} \sum_g q_{ig}^2$; ("handling cost")

Handling costs:

- can be rationalized through a "standard" production function: Technology
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Existence

Theorem

A non-trivial Supply and Demand Function equilibrium exists in any network such that every good is traded by at least 3 agents (=firms+consumer).

Key element of the proof:

- best reply coefficient matrices increasing in psd ordering;
- this also yields an algorithm to solve it (iterating the best reply).

Setting I

Firms

- N firms, each produces one good;
- M markets for M goods, $M < N$: some firms produce the same good;

- firms need specific goods as inputs - this defines the *input-output network* (exogenous);

Consumers

- continuum - **price taker representative consumer**
- consumers provide labor (L) and own the firms Alternative;
- competitive labor market: wage **taken as given** (normalized to 1).

Setting II - Parametric assumptions

Consumers: evaluate consumption bundles $\boldsymbol{c} = (c_1, \dots, c_N)$ using:

$$B_c^{-1}\boldsymbol{A}_c\boldsymbol{c} - \frac{1}{2}\boldsymbol{c}B_c^{-1}\boldsymbol{c} - L \Rightarrow D_c(\boldsymbol{p}_c) = \boldsymbol{A}_c - B_c\boldsymbol{p}_c$$

B_c pos.def.

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In this presentation, to simplify notation: $\omega_{ij} = 1$, $k_i = 1$.

The Game

The firms play a **simultaneous game** in which they commit to:

- **linear schedules**: $\boldsymbol{q}_i = (S_i, -(D_{ij})_{j \to i}, \ldots,) = B_i \boldsymbol{p}_i$;
- $\boldsymbol{p}_i = (p_j)$ s.t. j input or output of i;
- B_i symmetric positive semidefinite, corank 1;
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Their profits are:

$$\tilde{\pi}_i(B_i, B_{-i}) = \underbrace{p_i^* S_i}_{\text{Revenues}} - \underbrace{\sum_g p_g^* D_{ig}}_{\text{Intermediate input costs}} - \underbrace{\frac{1}{2} \sum_g D_{ig}^2}_{\text{Labor costs}}$$

where $\boldsymbol{p}^*$ are the prices that **solve the market clearing equations**:

$$\text{Demand for good } i = \text{Supply for good } i \quad \forall i$$

Details.

Solution

The market clearing equations are a linear system:

$$M\boldsymbol{p} = \boldsymbol{A}$$

where:

- $M = \sum_i \hat{B}_i + \hat{B}_c$, where $\hat{}$ represents **lifting**;
- $\boldsymbol{A}$ contains the intercepts of consumer demand.

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- $\boldsymbol{A}$ contains the intercepts of consumer demand.

Now we **partially solve** the system **fixing** the quantities traded by i:

- we obtain the **residual schedule** $\boldsymbol{q}_i^r(\boldsymbol{p}_i)$;

Solution

Crucially, the residual schedule:

- depends only on prices of goods traded by i;
- $\left(\frac{\partial \boldsymbol{q}_i^r}{\partial \boldsymbol{p}_i}\right)^{-1}$ is the **price impact** matrix;
- $\Lambda_i = [(M - \hat{B}_i)^{-1}]_i$ ($[\cdots]_i$ means "restricted to neighbors of" i).

Example

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  Example

Now the best reply problem becomes:

$$\max_{B_i} \pi_i \left(\boldsymbol{p}_i, \boldsymbol{q}_i^r(\boldsymbol{p}_i)\right)$$

But now payoff depends on B_i only through $\boldsymbol{p}_i$, so:

- "as if" firms optimize over $\boldsymbol{p}_i$ directly.

FOC

The FOCs yield an equation relating **matrices** of coefficients:

$$B_i = \Lambda_i^{-1} - \Lambda_i^{-1} \left(C_i + \Lambda_i^{-1} \right)^{-1} \Lambda_i^{-1}$$

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where:

- C_i coefficients under price taking;
- **strategic complementarity**: $B_j \uparrow \implies B_i \uparrow$ in psd ordering.

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Λ_i contains the "network effects". Example, in the supply chain:

$$\frac{\partial}{\partial p_U} D^r_{U1}(p_U) = -\overbrace{\left(\underbrace{\left(\frac{1}{B_c} + \frac{1}{B_{D1}+B_{D2}}\right)^{-1}}_{\text{Analogy with series/parallel resistors?}} + B_{U2}\right)}^{\text{(inverse) price impact (on output)}}$$

Related literature

Market power and efficiency (macro):

production networks **Acemoglu and Tahbaz-Salehi (WP)**, Grassi (WP), Kikkawa et al. (2020), Baqaee (2019), Baqaee and Farhi (2019, 2020), Pasten et al. (2018), Carvalho et. al (WP);

no input-output Pellegrino (2024), Azar and Vives (2021), Alviarez et. al (2023), Morlacco (2020);

Market power and efficiency in networks:

mergers Loertscher and Marx (WP), Hinnosaar ('19); Bimpikis et al. ('20), Hart and Tirole ('90), Salinger ('90);

bargaining Condorelli et al. ('17), Kotowski, Leister ('19), Manea ('18);

matching Hatfield et al. ('12), Fleiner et al. ('20), Fleiner et al. ('19);

Supply function competition/double auctions:

Supply function competition Klemperer and Meyer (1989), Green and Newbery (1992), Vives (2011);

Finance microstructure Kyle (1989), **Malamud and Rostek (2017)**;

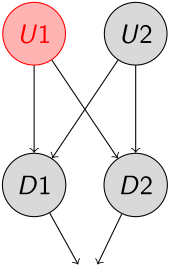
Auctions Ausubel et al. (2014), Woodward (WP).

General Oligopolistic Equilibrium:

Benassy (1988), Dierker and Grodhal (1999), Azar and Vives (2021)

back

Solution-example



In a supply chain with layers is sufficient to restrict attention to:

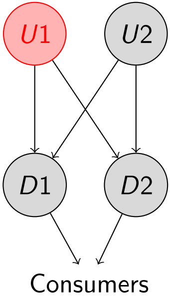
$$S_{Ui} = B_{Ui} p_U$$
$$D_{Di} = S_{Di} = B_{Di}(p_D - p_U)$$

The best reply of $U1$ to (D_{U2}, S_{D1}, S_{D2}) solves:

$$\max_{B_{U1}} \pi_{U1}(p_U(B_{U1}, \ldots), B_{U1} p_U(B_{U1}, \ldots))$$

back

Solution-example



Market clearing is:

$$\begin{cases} \boldsymbol{q}_{U1}(p_U) = B_{U1} p_U \quad \underbrace{(B_{D1}+B_{D2})(p_D - p_U)}_{\text{Demand from } D1,2} - \underbrace{B_{U2}(p_U)}_{\text{Supply of competitors}} \\ A_c - B_c p_D = \quad (B_{D1}+B_{D2})(p_D - p_U) \end{cases}$$

back

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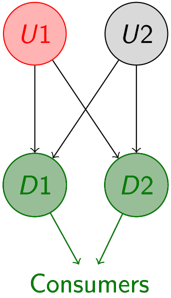

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back

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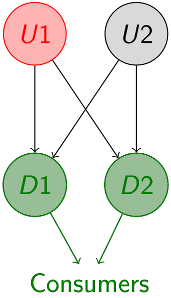

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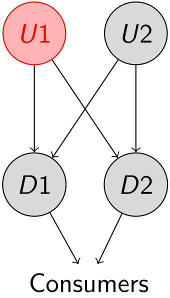

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where $D^r_{U1}(p_U)$ is the **residual demand**.

The slope depends on all downstream firms coefficients:

$$\frac{\partial}{\partial p_U} D^r_{U1}(p_U) = - \overbrace{\left(\underbrace{\left(\frac{1}{B_c} + \frac{1}{B_{D1} + B_{D2}} \right)^{-1} + B_{U2}}_{\text{Analogy with series/parallel resistors?}} \right)}^{\textbf{(inverse) price impact (on output)}}$$

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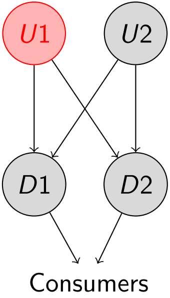

Now the best reply problem is:

$$\max_{B_{U1}} \pi_{U1}(p_U, D^r_{U1}(p_U))$$

back

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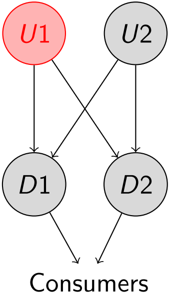

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back

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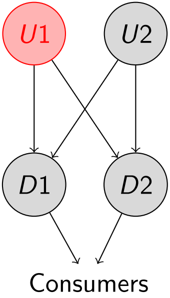

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Same FOC as a monopolist!

- Each firm sets preferred price;
- but it does so varying the slope;
- $\Longrightarrow$ it changes preferred prices of others.

back